

DPP-1

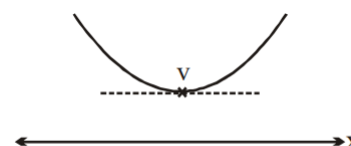
SOLUTIONS

1. Given $2x_1x_2 - x_1 = x_2 + 4$
 $2x_1x_2 - (x_1 + x_2) = 4$
 $2q + p = 4$
3. $D_1: b^2 - 4ac$ and $D_2: d^2 + 4ac$
Hence atleast one of either D_1 or D_2 is zero.
5. Given, $f(x) = x^2 + (2 - \lambda)x + \left(\lambda - \frac{3}{4}\right)$
Now, for a quadratic expression to become a perfect square, its $D = 0$
 $\Rightarrow (2 - \lambda)^2 = 4\left(\lambda - \frac{3}{4}\right) \Rightarrow 4 + \lambda^2 - 4\lambda = 4\lambda - 3$
 $\Rightarrow \lambda^2 - 8\lambda + 7 = 0 \Rightarrow (\lambda - 7)(\lambda - 1) = 0$
 $\therefore \lambda = 1, 7$
Hence, $(\lambda_1^2 + \lambda_2^2) = (1)^2 + (7)^2$
 $= 1 + 49 = 50$
10. $(k - 1)(k + 1)x^2 + (2k + 7)(k - 1)(k + 2)x + (2k + 7)(k - 2)(k + 1) = 0$
(A) Exactly one root is '0' and other root is finite. P.O.R. = 0 but S.O.R. $\neq 0$ Therefore $k = 2$.
(B) Both roots are zero means S.O.R. = P.O.R. = 0 Hence, $k = \frac{-7}{2}$ only
(C) Exactly one root infinity i.e. coefficient of $x^2 = 0$ but coefficient of $x \neq 0$ Therefore $k = -1$ only.
(D) Both roots are infinity i.e. coefficient of $x^2 =$ coefficient of $x = 0$ but constant term $\neq 0$
Therefore $k = 1$

DPP-2

SOLUTIONS

1. $r > 0; p < 0$ & $q < 0$. Now verify it.
3. We have, $(p - 3)x^2 - 2px + 3(p - 2) > 0$ is true for all $x \in \mathbb{R}$, so
 $p - 3 > 0$ (i)
and Disc. < 0 (ii)
must be satisfied simultaneously.
 $\therefore p > 3$ (1)
and $D < 0 \Rightarrow 4p^2 - 4(p - 3)(3p - 6) < 0$
 $f(x) = (p - 3)x^2 - 2px + 3(p - 2)$
 $\Rightarrow 2p^2 - 15p + 18 > 0$
 $\Rightarrow 2p^2 - 12p - 3p + 18 > 0$
 $\Rightarrow 2p(p - 6) - 3(p - 6) > 0$
 $\Rightarrow (2p - 3)(p - 6) > 0$
 $\Rightarrow p > 6$ or $p < \frac{3}{2}$ (2)
 $\therefore (1) \cap (2) \Rightarrow p \in (6, \infty)$
Hence, the smallest integral value of p equals 7.



5. $a < 0, c > 0$. Also, $\frac{-b}{2a} > 0 \Rightarrow b > 0$

So, $abc < 0$

Also, $f(-1) = a - b + c = 0$ and $f(3) = 9a + 3b + c = 0$

Clearly, $f\left(\frac{1}{3}\right) > 0 \Rightarrow \frac{a}{9} + \frac{b}{3} + c > 0$

$\Rightarrow a + 3b + 9c > 0$

Also $f\left(\frac{-1}{3}\right) > 0 \Rightarrow \frac{a}{9} - \frac{b}{3} + c > 0 \Rightarrow a - 3b + 9c > 0$

Now, verify alternatives

DPP-3

SOLUTIONS

1. With the condition, $c^2 = 4d$, the equation $x^2 - cx + d = 0$ has equal roots.

Each root $= \frac{c}{2}$.

As $\frac{c}{2}$ is a root of $x^2 - ax + b = 0$

$\therefore \frac{c^2}{4} - a \cdot \left(\frac{c}{2}\right) + b = 0$

$\Rightarrow \frac{4d}{4} - \frac{ac}{2} + b = 0 \Rightarrow b + d = \frac{ac}{2}$

$\Rightarrow 2(b + d) = ac$.

9. We have, $x^3 + 4x^2 + px + q = 0$ $\begin{matrix} \swarrow \alpha \\ \searrow \beta \\ \searrow \gamma \end{matrix}$ (i)

$x^3 + 6x^2 + px + r = 0$ $\begin{matrix} \swarrow \alpha \\ \searrow \beta \\ \searrow \delta \end{matrix}$ (ii)

(Subtract)

$2x^2 + (r - q) = 0$ $\begin{matrix} \swarrow \alpha \\ \searrow \beta \end{matrix}$

So, $\alpha + \beta = 0 \Rightarrow \gamma = -4$ and $\delta = -6$.

As, $x^2 + 2ax + 8c = 0$ $\begin{matrix} \swarrow \gamma \\ \searrow \delta \end{matrix}$

So, $\gamma + \delta = -2a \Rightarrow a = 5$; $\gamma\delta = 8c \Rightarrow c = 3$

Also, $\frac{\alpha\beta\gamma}{\alpha\beta\delta} = \frac{-q}{-r} \Rightarrow \frac{2}{3} = \frac{q}{r} \Rightarrow 3q = 2r$